On the forces in single-ended and push-pull electret transducers

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The equations for the electromechanical force conversion in single-ended and push-pull electret transducers are derived. Traditionally, the charge distribution has been modeled as a concentrated layer at an arbitrary distance from the surface of the dielectric. For the purpose of this analysis, a negative charge is assumed to be evenly distributed throughout the dielectric. The membrane has a conductive coating in which a positive charge is induced, giving an overall dipole charge. The resulting formulas are used to derive the voltage sensitivity of a microphone and the equivalent electrical circuit for the electromechanical transduction part of a microphone or loudspeaker. An equivalent external polarizing voltage is then derived that would produce the same driving force in a conventional electrostatic loudspeaker without a stored charge. The condition for the static stability of a circular electret membrane is also determined.
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I. INTRODUCTION

Since the electret microphone was introduced,[1,2] it has almost completely replaced all other types in the field of communications due to its robustness, reliability, and near studio quality. However, the application of an electret loudspeaker has remained largely elusive and electrostatic loudspeakers still invariably use an external polarizing supply in order to create a monopole charge in the conductive layer of the central membrane. The advantage of a monopole charge is that for a given membrane charge, the field does not vary with electrode separation. Electret membranes usually have a conductive coating which, while increasing the charge storage stability, produces a dipole charge. That is, a charge of the opposite polarity to that contained by the membrane is induced in the metallic coating. As a result, for a given membrane charge, the field decreases with increasing electrode separation. This is not such a major issue in the case of a microphone, where the electrode separation is typically very small, but in the case of a loudspeaker, wider separation is needed in order to achieve the volume acceleration that, in turn, is required to give a reasonable sound pressure level at some distance from it. Hence, membranes with a monopole charge have been investigated.[3,4] In recent years, electret membrane technology has been developing in terms of both the charge storage stability and maximum charge density. Porous membranes,[5,6] in particular, have received much attention and store enough charge to compensate for the dipole effect, having internal potentials of up to 500–1000 V. Hence, only dipole membranes are considered in this paper.

In accordance with Paschen's curve,[7] dipole charges are induced in the pores (pore size greater than 1 μm) by breaking down air inside the pores during the charging process. Both dipole and monocharges can be trapped in a porous electret dielectric. The pore size and pore density influences the relative density of dipole and monocharges. In general, the piezoelectric effect is found in porous electrets when the pore size is a few micrometers. On the other hand, reducing the size and density of the pores increases the amount of monocharge. In this paper, it is assumed that the monocharge dominates in a nanoporous material and, although the exact charge distribution is usually unknown, a possible distribution is shown in Fig. 1. It is likely that some charge will be lost at the outer faces, especially the one adjoining the conductive coating, where there will be some recombination with the positive charge induced in the coating. Hence, there will be a peak somewhere near the middle. However, in order to approximate this, a uniform charge distribution is assumed throughout the membrane.

For loudspeakers, another issue is that of space. Most pressure (or monopole) microphones convert pressure into mechanical displacement which in turn produces electrical charge displacement. Hence, they operate in the displacement controlled frequency range below the fundamental mechanical resonance. Loudspeakers, on the other hand, have to produce volume acceleration. Hence, they operate in the acceleration controlled frequency range above the fundamental mechanical resonance. Membrane loudspeakers have very little moving mass. In fact, the mass of the acoustic radiation load is usually greater than that of the membrane. In order to achieve a low enough fundamental resonance with such a small mass requires high mechanical compliance. Hence, a small enclosure is not an option. However, recent technological developments, such as activated carbon,[8,9] are moving in the right direction. Alternatively, a membrane loudspeaker can be used without an enclosure (i.e., as a dipole), but this requires a large diaphragm area in order to minimize rear-wave cancellation at low frequencies.

The theory of constant-charge push-pull electrostatic transducers with external polarizing supplies has long been established,[10] but there is little if any literature on the theory of push-pull electret transducers, although the force conver-

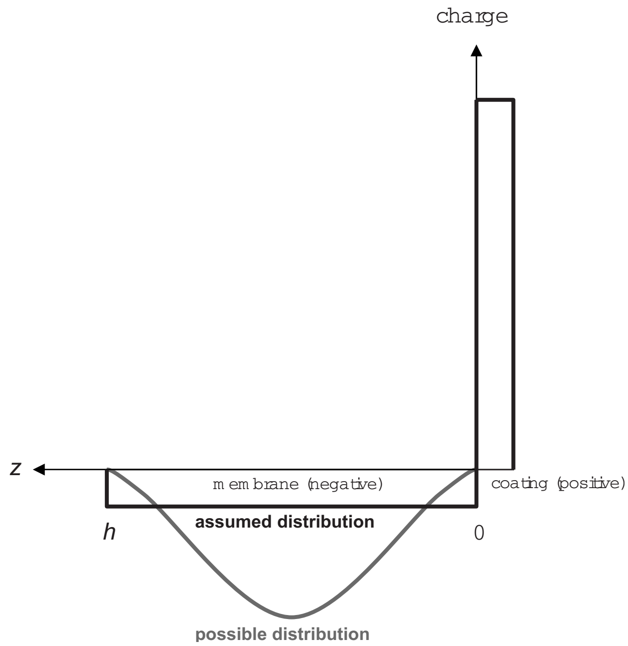

FIG. 1. Assumed charge distribution in membrane.

sion in single-ended electret transducer due to Sessler[11] is reproduced here by degenerating a push-pull model. Two push-pull configurations are analyzed in this paper. The first has a floating membrane, with just leakage paths between the conductive membrane coatings and the outer electrodes, represented by high-value resistors. Under dynamic conditions, these are ignored so that the configuration is treated as fully floating. However, under static conditions, the membrane is effectively grounded. Hence, the grounded membrane forms the basis of the second configuration to be analyzed. The forces and charges are also derived for a single-ended configuration, as well as a method to evaluate the stored charge density. An equivalent electrical circuit is developed which is intended as a basis for simulation, since sound radiation from membranes has already been analyzed in depth.[12–15] Finally, the condition for the static stability of a circular electret transducer is determined and compared with that of Streng[12] for a nonelectret transducer. The criteria for the stability of capacitance microphones have also been studied numerically by Warren,[16] although no closed-form conditional equations for specific geometries were presented.

II. PUSH-PULL ELECTRET TRANSDUCER DESCRIPTION

A push-pull configuration with a semifloating membrane is shown in Fig. 2. The two resistors R have very high values or could just represent leakage paths. Under quiescent conditions, with zero input signal, the membrane behaves as though it is strapped to the outer electrodes and hence the static charges assume the same values as in the case of a grounded membrane. This is the most stable steady state condition and also applies to very low frequencies, where half the input voltage is effectively connected between the membrane's conductive coatings and each outer electrode. However, when driven by a signal voltage across the outer electrodes at medium to high frequencies, the membrane behaves

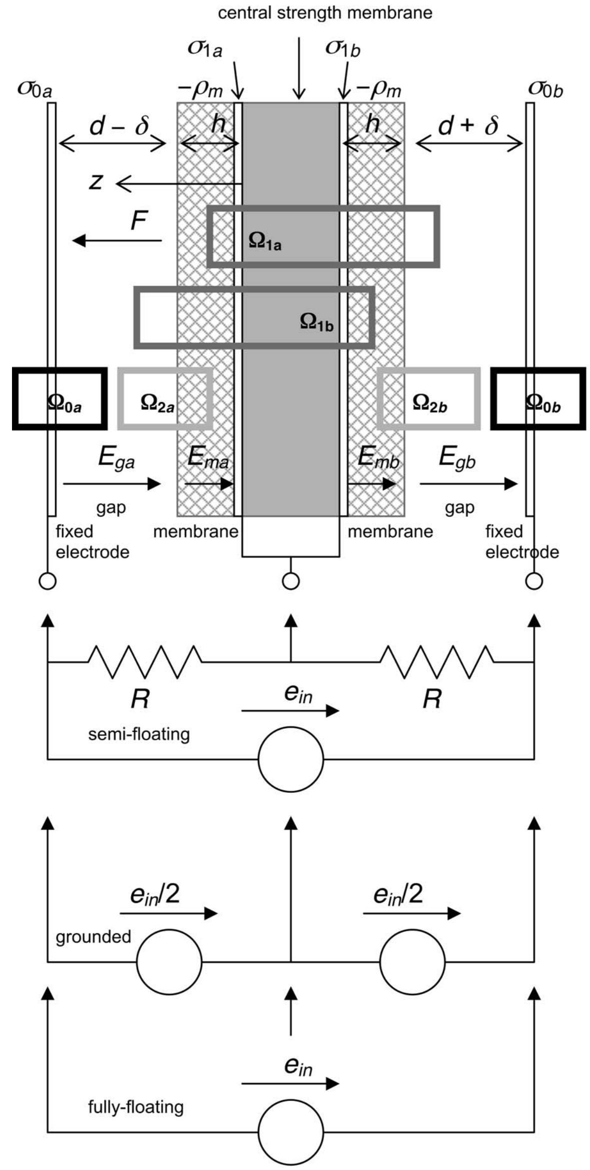

FIG. 2. Push-pull configuration with semifloating, grounded, and fully-floating membrane.

as though its conductive coatings are isolated and fully floating. This has a significant effect on the linearity of the membrane displacement.

The configuration in Fig. 2 is similar to one previously proposed by Atoji and Aoi,[17] except that their configuration has just one single charged membrane. Here, two are provided in order to maintain symmetry and to prevent the membrane from undergoing a permanent static displacement to one side, which would limit the maximum excursion. E_{ga} and E_{gb} are the electric field strengths in each of the gaps either side of the membrane, which actually comprises two membranes, each attached to a central strength membrane. Since the central strength membrane contains no charge, it does not contribute to the electric field and is thus assumed to be infinitesimally thin in the following force derivation. E_{ma} and E_{mb} are the electric field strengths within the two membranes, respectively, which are both assumed to contain an evenly distributed negative charge per unit volume (or volume charge density) $-\rho_m$. The inner electrode layers have surface charge densities σ_{1a} and σ_{1b}. The outer plates have surface charge densities σ_{0a} and σ_{0b}. The relative permittivities of the membrane material and acoustic medium in the gaps are ε_r and ε_{r1}, respectively. If the gaps contain air, then ε_{r1} can be assumed to be unity.

III. GAUSS'S LAW

Gauss's law[18] states that the difference in electrical flux is equal to the net charge inside the volume. Applying this to the cylinders Ω_{0a} and Ω_{0b} gives

$$\varepsilon_0\varepsilon_{r1}E_{ga} = \sigma_{0a}, \tag{1}$$

$$-\varepsilon_0\varepsilon_{r1}E_{gb} = \sigma_{0b}. \tag{2}$$

It is assumed that the size of the electrodes is very large in comparison to the distance between them so that the field is perpendicular to the electrodes and the sides of the cylinders give zero flux. The tops of the cylinders are at constant distances from the outer electrodes, also the values of the electric fields are constant on the surfaces. Hence the fluxes are trivial to compute. Outside of the system, the field is assumed to be zero. The fluxes through the cylinders Ω_{2a} and Ω_{2b} are a little more complicated to calculate, since the membrane has a constant charge density:

$$\varepsilon_0\varepsilon_r E_{ma}(z) - \varepsilon_0\varepsilon_{r1}E_{ga} = -\rho_m(h-z), \tag{3}$$

$$\varepsilon_0\varepsilon_{r1}E_{gb} - \varepsilon_0\varepsilon_r E_{mb}(z) = -\rho_m(h+z). \tag{4}$$

Substituting Eq. (1) into Eq. (3) yields

$$E_{ma}(z) = \frac{\sigma_{0a} - (1 - z/h)\sigma_m}{\varepsilon_0\varepsilon_r} \tag{5}$$

and likewise substituting Eq. (2) into Eq. (4) yields

$$E_{mb}(z) = \frac{-\sigma_{0b} + (1 + z/h)\sigma_m}{\varepsilon_0\varepsilon_r}, \tag{6}$$

where $\sigma_m = \rho_m h$ is the constant charge per unit area (or surface charge density) of the membrane. Similarly, the fluxes through the cylinders Ω_{1a} and Ω_{1b} can be calculated as follows:

$$\varepsilon_0\varepsilon_{r1}E_{gb} - \varepsilon_0\varepsilon_r E_{ma}(z) = \sigma_{1a} + \sigma_{1b} - \rho_m(z+h), \tag{7}$$

$$\varepsilon_0\varepsilon_r E_{mb}(z) - \varepsilon_0\varepsilon_{r1}E_{ga} = \sigma_{1a} + \sigma_{1b} + \rho_m(z-h). \tag{8}$$

Substituting Eqs. (2) and (5) into Eq. (7) yields the following relation between the surface charge densities:

$$\sigma_{0a} + \sigma_{0b} + \sigma_{1a} + \sigma_{1b} - 2\sigma_m = 0. \tag{9}$$

Likewise, substituting Eqs. (1) and (6) into Eq. (8) yields the same relation between the surface charge densities

$$\sigma_{0a} + \sigma_{0b} + \sigma_{1a} + \sigma_{1b} - 2\sigma_m = 0. \tag{10}$$

Equations (1), (2), (5), and (6) give the functional forms of the electric fields, where E_{ga} and E_{gb} are constants and E_{ma} and E_{mb} are linear functions of z. The unknowns here are the surface charge densities σ_{0a}, σ_{0b}, σ_{1a}, and σ_{1b}. Their values depend on how the electrodes have been connected to each other.

IV. KIRCHHOFF'S LOOP RULE

A. Floating membrane

Now, let a potential (voltage) e_{in} be connected between the outermost electrodes, as shown in Fig. 2. In the fully floating state, the values of the resistors R are assumed to be infinite. This represents dynamic conditions at medium to high frequencies, where the impedance due to the interelectrode capacitance is much smaller than the impedance of the resistors. Using the loop equation (the direction of the electric fields is from left to right, but the integration from right to left) gives

$$\begin{aligned} -e_{in} &= -\int_{-h-d-\delta}^{-h} E_{gb}dz - \int_{-h}^{0} E_{mb}(z)dz - \int_{0}^{h} E_{ma}(z)dz \\ &\quad - \int_{h}^{h+d-\delta} E_{ga}dz \\ &= \frac{(\varepsilon_r(d+\delta) + \varepsilon_{r1}h)\sigma_{0b} - (\varepsilon_r(d-\delta) + \varepsilon_{r1}h)\sigma_{0a}}{\varepsilon_0\varepsilon_r\varepsilon_{r1}}. \end{aligned} \tag{11}$$

where the electric fields E_{gb}, E_{mb}, E_{ma}, and E_{ga} are given by Eqs. (2), (6), (5), and (1), respectively.

B. Grounded membrane

At very low frequencies, the impedance of the resistors R in Fig. 2 is much smaller than that of the interelectrode capacitance. Hence, half of the input voltage is connected across each resistor

$$\begin{aligned} -\frac{e_{in}}{2} &= -\int_{-h-d-\delta}^{-h} E_{gb}dz - \int_{-h}^{0} E_{mb}(z)dz \\ &= \frac{2(\varepsilon_r(d+\delta) + \varepsilon_{r1}h)\sigma_{0b} - \varepsilon_{r1}h\sigma_m}{2\varepsilon_0\varepsilon_r\varepsilon_{r1}}. \end{aligned} \tag{12}$$

$$\begin{aligned} -\frac{e_{in}}{2} &= -\int_{0}^{h} E_{ma}(z)dz - \int_{h}^{h+d-\delta} E_{ga}dz = \\ &\quad -\frac{(\varepsilon_r(d-\delta) + \varepsilon_{r1}h)\sigma_{0a} - \varepsilon_{r1}h\sigma_m}{2\varepsilon_0\varepsilon_r\varepsilon_{r1}}. \end{aligned} \tag{13}$$

where the electric fields E_{gb}, E_{mb}, E_{ma}, and E_{ga} are given by Eqs. (2), (6), (5), and (1) respectively.

V. SENSITIVITY AS A MICROPHONE

The voltage sensitivity is found by differentiating Eq. (11) with respect to displacement δ,

$$\Delta e_{in} = -\frac{\sigma_{0a} + \sigma_{0a}}{\varepsilon_0\varepsilon_{r1}}\Delta\delta. \tag{14}$$

For quiescent conditions ($\delta=0$, $e_{in}=0$), Eqs. (12) and (13) give

$$\sigma_{0a} = \sigma_{0b} = \frac{\varepsilon_{r1} h \sigma_m}{2(\varepsilon_r d + \varepsilon_{r1} h)}. \quad (15)$$

Hence, the linearized voltage sensitivity versus displacement of a floating push-pull microphone is defined by

$$\Delta e_{in} = -\frac{h\sigma_m}{\varepsilon_0(\varepsilon_r d + \varepsilon_{r1} h)} \Delta\delta. \quad (16)$$

The voltage sensitivity of a single-ended microphone is just half this value.

VI. CALCULATION OF THE NET FORCE AS A FUNCTION OF ELECTRODE CHARGES

The total energy W of the electric field can be calculated by integrating the dot product of the flux D and electric field E over the volume V [where $V = 2(d+h)S$ and S is the surface area of the membrane] as follows:

$$W = \frac{1}{2}\int_V \bar{D} \cdot \bar{E} dV = \frac{S}{2}\int \bar{D} \cdot \bar{E} dz, \quad (17)$$

where

$$\bar{D} = \begin{cases} \varepsilon_0 \varepsilon_{r1} \bar{E}_{gb}, & -h - d - \delta \le z < -h \\ \varepsilon_0 \varepsilon_r \bar{E}_{mb}, & -h \le z \le 0 \\ \varepsilon_0 \varepsilon_r \bar{E}_{ma}, & 0 \le z \le h \\ \varepsilon_0 \varepsilon_{r1} \bar{E}_{ga}, & h \le z < h + d - \delta. \end{cases} \quad (18)$$

Hence,

$$W = \frac{S}{2}\varepsilon_0\left(\varepsilon_{r1}\int_{-h-d-\delta}^{-h} E_{gb}^2 dz + \varepsilon_r \int_{-h}^{0} E_{mb}^2 dz + \varepsilon_r \int_0^h E_{ma}^2 dz + \varepsilon_{r1}\int_h^{h+d-\delta} E_{ga}^2 dz\right), \quad (19)$$

where the electric fields E_{gb}, E_{mb}, E_{ma}, and E_{ga} are given by Eqs. (2), (6), (5), and (1), respectively. In this case, the energy density of the system will depend on δ, and there will be a force proportional to the charge of the membrane. The net force F acting on the membrane is then given by

$$F = \frac{\partial W}{\partial \delta} = \frac{S(\sigma_{0b}^2 - \sigma_{0a}^2)}{2\varepsilon_0 \varepsilon_{r1}}. \quad (20)$$

VII. CHARGE DENSITIES

A. Floating membrane

Solving Eqs. (9) and (11) simultaneously for the charge densities σ_{0a} and σ_{0b} yields

$$\sigma_{0a} = \frac{(\varepsilon_r(d+\delta) + \varepsilon_{r1} h)(2\sigma_m - (\sigma_{1a} + \sigma_{1b})) + \varepsilon_0 \varepsilon_r \varepsilon_{r1} e_{in}}{2(\varepsilon_r d + \varepsilon_{r1} h)}, \quad (21)$$

$$\sigma_{0b} = \frac{(\varepsilon_r(d-\delta) + \varepsilon_{r1} h)(2\sigma_m - (\sigma_{1a} + \sigma_{1b})) - \varepsilon_0 \varepsilon_r \varepsilon_{r1} e_{in}}{2(\varepsilon_r d + \varepsilon_{r1} h)}. \quad (22)$$

It can be seen that the charge on the outer electrodes varies as the membrane moves from its central position, due to the charge movement between them via the voltage source. However, their sum is equal to the total charge density on the membrane as follows:

$$\sigma_{0a} + \sigma_{0b} = 2\sigma_m - (\sigma_{1a} + \sigma_{1b}) \quad (23)$$

and this remains constant. There may be similar charge movements between the charges σ_{1a} and σ_{1b} on the conductive coatings of the membranes, but this does not matter because it is the total membrane charge density that governs Eqs. (21) and (22).

B. Grounded membrane

Rearranging the loop rule of Eq. (13) yields an expressions for the charge density σ_{0a} as follows:

$$\sigma_{0a} = \frac{\varepsilon_{r1}(h\sigma_m + \varepsilon_0 \varepsilon_r e_{in})}{2(\varepsilon_r(d-\delta) + \varepsilon_{r1} h)}. \quad (24)$$

Rearranging the loop rule of Eq. (12) yields an expressions for the charge density σ_{0b} as follows:

$$\sigma_{0b} = \frac{\varepsilon_{r1}(h\sigma_m - \varepsilon_0 \varepsilon_r e_{in})}{2(\varepsilon_r(d+\delta) + \varepsilon_{r1} h)}. \quad (25)$$

In this case, the sum of the charges on the outer electrodes is not independent of the membrane position.

C. Single ended

A single-ended transducer can be created by removing all of the structure to the right of the central strength membrane in Fig. 2 and replacing the resistor R on the right with a short circuit. In this case, $\sigma_{0b} = \sigma_{1b} = E_{gb} = E_{mb} = 0$ and Eq. (7) becomes

$$-\varepsilon_0 \varepsilon_r E_{ma}(z) = \sigma_{1a} - \rho_m z. \quad (26)$$

Substituting Eq. (5) in Eq. (26) yields

$$\sigma_{0a} + \sigma_{1a} - \sigma_m = 0. \quad (27)$$

In the quiescent state, where $e_{in} = 0$ and $\delta = 0$, Eq. (24) becomes

$$\sigma_{0a} = \frac{\varepsilon_{r1} h \sigma_m}{2(\varepsilon_r d + \varepsilon_{r1} h)}. \quad (28)$$

Referring to Fig. 3, it can be seen that as the gap width d is increased to infinity, σ_{0a} tends to zero, but σ_{1a} becomes asymptotically equal and opposite to the membrane charge $-\sigma_m$. Hence, the membrane, together with its conductive coating, forms a dipole which is its natural stable state. When $d = 0$, the charge is shared equally between the two electrodes such that each holds half the value of the membrane charge.

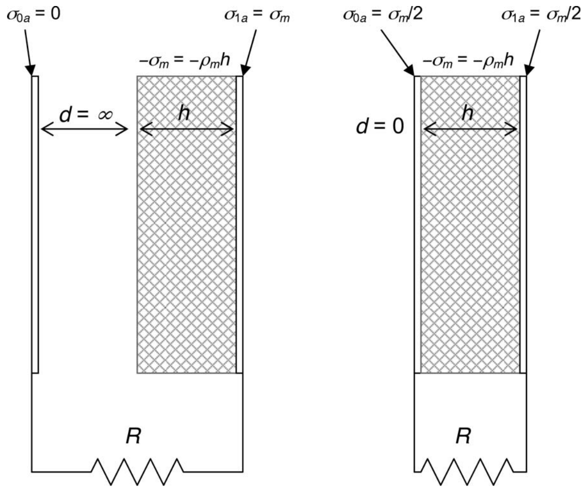

FIG. 3. Induced electrode charge at $d=0$ and $d=\infty$.

VIII. CALCULATION OF THE NET FORCE AS A FUNCTION OF VOLTAGE

A. Floating membrane

The net force is calculated from Eqs. (20)–(22). The charges σ_{1a} and σ_{1b} are the quiescent ones, in which state the membrane can be assumed to be grounded. They are taken from Eq. (9) as follows:

$$\sigma_{1a} + \sigma_{1b} = 2\sigma_m - \sigma_{0a} - \sigma_{0b}, \tag{29}$$

which, after inserting σ_{0a} and σ_{0b} from Eqs. (24) and (25), respectively, gives

$$\sigma_{1a} + \sigma_{1b} = 2\sigma_m - \frac{\varepsilon_{r1} h \sigma_m}{\varepsilon_r d + \varepsilon_{r1} h}, \tag{30}$$

which is assumed to be constant under dynamic conditions. After inserting Eqs. (21), (22), and (30) into Eq. (20), the net force F is then given by

$$F = -\frac{\varepsilon_r \varepsilon_{r1} h S \sigma_m}{2(\varepsilon_r d + \varepsilon_{r1} h)^2} e_{in} - \frac{\varepsilon_r \varepsilon_{r1} h^2 S \sigma_m^2}{2\varepsilon_0 (\varepsilon_r d + \varepsilon_{r1} h)^3} \delta. \tag{31}$$

There are two components to the force, which is perfectly linear. The first is due to the input voltage source and the second is due to a "negative stiffness" or the static attraction due to the membrane charge. Alternatively, Eq. (31) may be written as

$$F = \chi e_{in} + \kappa \delta, \tag{32}$$

where χ is the voltage-force conversion factor given by

$$\chi = \left(\frac{C_E}{C_M + C_G}\right) \frac{S\sigma_m}{2d} \approx \frac{\varepsilon_{r1} h S \sigma_m}{2\varepsilon_r d^2}, \quad d \gg \frac{\varepsilon_{r1} h}{\varepsilon_r}, \tag{33}$$

where C_E is the static capacitance between the outer electrodes when the membrane is blocked, which is given by

$$C_E = \frac{C_M C_G}{C_M + C_G}, \tag{34}$$

where C_M is the total capacitance of the two membranes given by

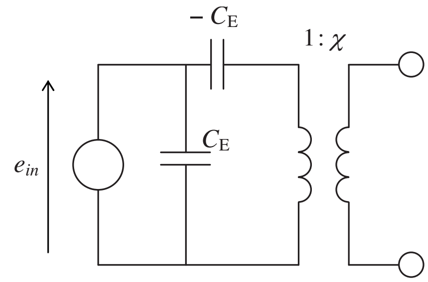

FIG. 4. Equivalent electrical circuit.

$$C_M = \frac{\varepsilon_0 \varepsilon_r S}{2h}, \tag{35}$$

and C_G is the total capacitance of the two gaps given by

$$C_G = \frac{\varepsilon_0 \varepsilon_{r1} S}{2d} \tag{36}$$

and κ is the negative stiffness given by

$$\kappa = \frac{C_E}{2(C_M + C_G)^2} \left(\frac{S\sigma_m}{d}\right)^2 \approx \frac{\varepsilon_{r1} h^2 S \sigma_m^2}{2\varepsilon_0 \varepsilon_r^2 d^3}, \quad d \gg \frac{\varepsilon_{r1} h}{\varepsilon_r}. \tag{37}$$

The equivalent electrical circuit is shown in Fig. 4, which can be used as part of a larger model including the dynamic impedance of the membrane and surrounding acoustic system. The force is plotted against displacement in Fig. 5 using Eq. (31) with $e_{in}=0$. It is interesting to compare this configuration with that of a nonelectret transducer[13] with an external polarizing voltage E_P, for which

$$\chi = \frac{E_P C_{ED}}{d} = \frac{\varepsilon_0 S E_P}{d^2}. \tag{38}$$

Hence, the external polarizing voltage for the equivalent nonelectret transducer is given by

$$E_P = \frac{S\sigma_m}{2(C_M + C_G)} \approx \frac{\sigma_m h}{2\varepsilon_0 \varepsilon_r}, \quad d \gg \frac{\varepsilon_{r1} h}{\varepsilon_r}. \tag{39}$$

Inserting σ_m from this equation into Eq. (31) indeed transforms it into Eq. (3.13) of Borwick[19] for the forces in a non-electret transducer.

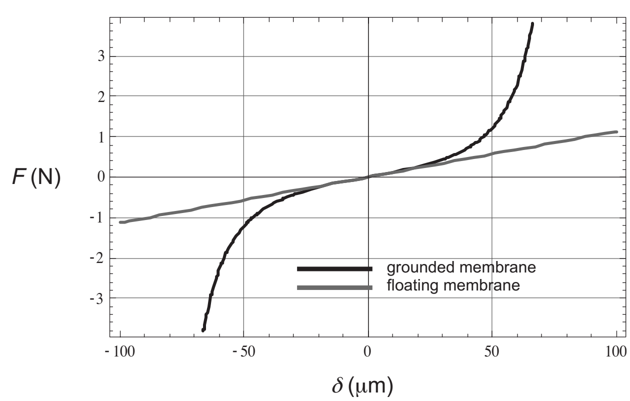

FIG. 5. Linearity of force vs displacement of push-pull transducer with input shorted where $S=\pi a^2$, where $a=15$ mm, $d=200$ μm, $h=12$ μm, $\sigma_m=14.1$ mC/m^2, $\varepsilon_0=8.85$ pF/m, $\varepsilon_r=12$, and $\varepsilon_{r1}=1$.

B. Grounded membrane

After inserting Eqs. (24) and (25) into Eq. (20), the net force F is then given by

$$F=\frac{\varepsilon_{r1}S}{8\varepsilon_0}\left\{\left(\frac{\varepsilon_0\varepsilon_r e_{in}-h\sigma_m}{\varepsilon_r(d+\delta)+\varepsilon_{r1}h}\right)^2-\left(\frac{\varepsilon_0\varepsilon_r e_{in}+h\sigma_m}{\varepsilon_r(d-\delta)+\varepsilon_{r1}h}\right)^2\right\}. \quad (40)$$

Not surprisingly, this is the resultant force of two single-ended transducers mechanically coupled back to back. While the voltage to force conversion of this configuration is perfectly linear when $\delta=0$ (unlike a single-ended configuration), the force is nonlinear with displacement, as can be seen in Fig. 5, and it increases asymptotically as the membrane approaches each electrode where it is singular. The small-signal linearized expressions for the negative stiffness κ and voltage-force conversion factor χ can be obtained as follows:

$$\kappa=-\left.\frac{\partial F}{\partial\delta}\right|_{\delta=0,e_{in}=0}=\frac{\varepsilon_r\varepsilon_{r1}h^2S\sigma_m^2}{2\varepsilon_0(\varepsilon_r d+\varepsilon_{r1}h)^3}, \quad (41)$$

$$\chi=\left.\frac{\partial F}{\partial e_{in}}\right|_{\delta=0,e_{in}=0}=\frac{\varepsilon_r\varepsilon_{r1}hS\sigma_m}{2(\varepsilon_r d+\varepsilon_{r1}h)^2}, \quad (42)$$

and these are the same as in Eq. (31) for the fully floating configuration.

C. Single ended

It can be shown that the net force F is then given by

$$F=\frac{\varepsilon_{r1}S}{8\varepsilon_0}\left(\frac{2\varepsilon_0\varepsilon_r e_{in}+h\sigma_m}{\varepsilon_r(d+\delta)+\varepsilon_{r1}h}\right)^2 =\frac{\varepsilon_{r1}S}{2(\varepsilon_r(d+\delta)+\varepsilon_{r1}h)^2}\left(\frac{h^2\sigma_m^2}{4\varepsilon_0}+\varepsilon_r h\sigma_m e_{in}+\varepsilon_0\varepsilon_r^2e_{in}^2\right). \quad (43)$$

This equation is the same as that due to Sessler,[11] except that σ_m was multiplied by 2 in Sessler's model where a concentrated charge layer was assumed to exist at the interface between the membrane and the gap. It can be seen that there are three terms in the bracket. The first is the static force with zero input voltage. The other two are voltage dependent terms, the first of which is linear and the second, which depends upon the square of the input voltage, is nonlinear. It can also be seen that the force is nonlinear with displacement δ. Ignoring second order terms, the following linearized small-signal equation can be written as

$$F=\chi e_{in}+\kappa\delta, \quad (44)$$

where χ is the voltage to force conversion factor given by

$$\chi=\left(\frac{C_E}{C_M+C_G}\right)\frac{S\sigma_m}{2d}\approx\frac{\varepsilon_{r1}hS\sigma_m}{2\varepsilon_r d^2},\quad d\gg\frac{\varepsilon_{r1}h}{\varepsilon_r} \quad (45)$$

and C_E is the static capacitance between the outer electrodes when the membrane is blocked, which is given by

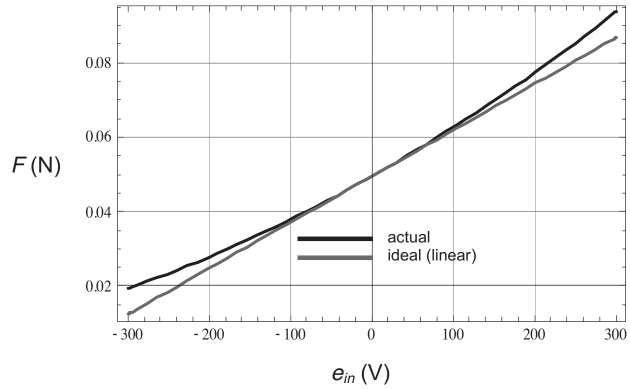

FIG. 6. Linearity of force vs voltage of single-ended transducer with membrane blocked where $S=\pi a^2$, where $a=15$ mm, $d=200$ μm, $h=12$ μm, $\sigma_m=14.1$ mC/m^2, $\varepsilon_0=8.85$ pF/m, $\varepsilon_r=12$, and $\varepsilon_{r1}=1$.

$$C_E=\frac{C_MC_G}{C_M+C_G}, \quad (46)$$

where C_M is the capacitance of the membrane given by

$$C_M=\frac{\varepsilon_0\varepsilon_r S}{h}, \quad (47)$$

and C_G is the capacitance of the gap given by

$$C_G=\frac{\varepsilon_0\varepsilon_{r1}S}{d}, \quad (48)$$

and κ is the negative stiffness. A small-signal linear approximation for κ is given by

$$\kappa=\frac{C_E}{4(C_M+C_G)^2}\left(\frac{S\sigma_m}{d}\right)^2\approx\frac{\varepsilon_{r1}h^2S\sigma_m^2}{4\varepsilon_0\varepsilon_r^2d^3},\quad d\gg\frac{\varepsilon_{r1}h}{\varepsilon_r}. \quad (49)$$

The equivalent electrical circuit is the same as that shown in Fig. 4 for a push-pull transducer. The force versus voltage characteristic of the blocked membrane is plotted in Fig. 6 using Eq. (43) with $\delta=0$. Also shown is the ideal linear case with the e_{in}^2 term omitted. The force versus displacement characteristic of the membrane with the electrodes shorted is plotted in Fig. 7 using Eq. (43) with $e_{in}=0$. Also shown is the ideal linear case using Eq. (49) added to the static force from Eq. (43) with $e_{in}=\delta=0$.

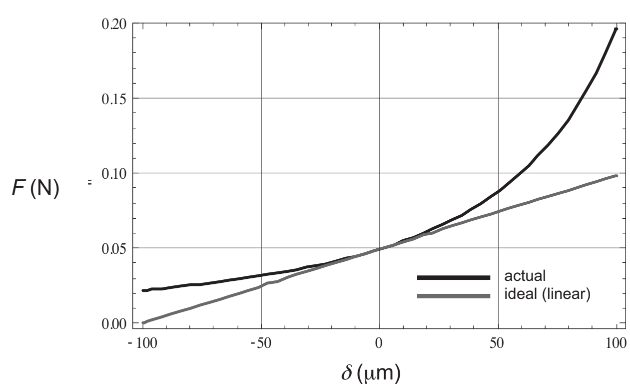

FIG. 7. Linearity of force vs displacement of single-ended transducer with input shorted and same parameters as in Fig. 6

D. Calculation of the net charge

In the case of a single-ended transducer, one can calculate the net stored charge by applying a reverse voltage such that the membrane displacement is reduced and then increasing it further until the displacement is about to increase again. Let the turning point occur at voltage e_0. Then differentiating Eq. (43) with respect to e_{in} gives

$$\left.\frac{dF}{de_{in}}\right|_{e_{in}=e_0} = \frac{\varepsilon_{r1}S(\varepsilon_r h\sigma_m + 2\varepsilon_0\varepsilon_r^2 e_{in})}{2(\varepsilon_r(d+\delta)+\varepsilon_{r1}h)^2} = 0, \quad (50)$$

so that the surface charge density is given by

$$\sigma_m = -\frac{2\varepsilon_0\varepsilon_r e_0}{h}. \quad (51)$$

By inserting this expression for σ_m in Eq. (43), it can be shown that $F=0$ when $e_{in}=e_0$, so there should be zero displacement at this point.

IX. STABILITY OF A CIRCULAR MEMBRANE

For a circular membrane, the following static wave equation can be written, ignoring inertia and external loading,

$$T\left(\frac{\partial^2}{\partial w^2}+\frac{1}{w}\frac{\partial}{\partial w}\right)\eta(w) - \frac{\kappa}{\pi a^2}\eta(w) = 0, \quad \eta(a) = 0, \quad (52)$$

where η is the membrane deflection, w is the radial ordinate, T is the tension, and a is the radius. The solution is given by

$$\eta(w) = \begin{cases} 0, & a\sqrt{\kappa/(\pi a^2 T)} \neq \alpha_n \\ CJ_0(\alpha_n w/a), & a\sqrt{\kappa/(\pi a^2 T)} = \alpha_n, \end{cases} \quad (53)$$

where α_n is the nth zero of J_0, C is an arbitrary constant, and the square-root term is the static wave number. Hence, for stability

$$T > \frac{\kappa}{\pi\alpha_1^2}, \quad (54)$$

where $\alpha_1=2.4048$. Using Eq. (37) for κ, where the area is given by $S=\pi a^2$, and assuming $d \gg \varepsilon_{r1}h/\varepsilon_r$ give

$$T > \frac{a^2\varepsilon_{r1}h^2\sigma_m^2}{2\alpha_1^2\varepsilon_0\varepsilon_r^2 d^3}, \quad d \gg \frac{\varepsilon_{r1}h}{\varepsilon_r}, \quad (55)$$

for a push-pull transducer. In the case of a single-ended transducer, this value of tension is halved. Inserting σ_m from Eq. (39) into Eq. (55) gives

$$T > \frac{2a^2\varepsilon_0\varepsilon_{r1}E_P^2}{\alpha_1^2 d^3}, \quad (56)$$

which is Streng's equation[12] for the stability of a nonelectret push-pull electrostatic transducer. It can be seen that as the charge density is increased, the tension also has to be increased in order to maintain stability. However, the fundamental resonant frequency of the membrane has to be set low enough to give the desired bandwidth and this is determined by the system stiffness. The system stiffness is that due to the membrane (which is proportional to its tension) less the negative stiffness due to the electrostatic force. As the charge density is increased, the difference between these two forces becomes an ever smaller proportion of the membrane stiffness. Hence, a small relaxation in its tension, which can easily occur due to age or environmental conditions, can lead to the membrane becoming attached to either of the electrodes. Some solutions have been proposed,[20] which involve increasing the stiffness of the membrane by adding "springs" between it and the electrodes. Unfortunately, these add nonlinearity to an otherwise linear transducer. Inevitably, they restrict the maximum displacement and hence also the maximum sound pressure. On the other hand, they can enable new forms such as flexible loudspeakers.[21]

X. DISCUSSION OF THE RESULTS

In all of the configurations considered, there is a near field effect whereby for very small electrode separations (i.e., $d < h\varepsilon_{r1}/\varepsilon_r$), the force is relatively independent of the separation. However, such a small separation is not such a practical proposition for a loudspeaker since the membrane would not be able to move far enough to produce a reasonable sound pressure. At larger distances, these expressions reduce to simpler far-field expressions in which the driving force is proportional to the inverse square of the separation and the negative stiffness is proportional to the inverse cube of the separation.

The floating push-pull electret transducer is utterly linear with both input voltage and displacement, as can be seen from Fig. 5, because the charges are kept constant by virtue of the floating membrane. However, it should be noted that under static conditions, as well as at frequencies below $f = 1/(2\pi RC_E)$, this configuration behaves as though the membrane is grounded due to charge leakage. Even if there are no physical resistors R connected to the conductive coatings, as shown in Fig. 2, those resistors will be replaced by charge leakage paths so that the static deflection is still nonlinear, as also shown in Fig. 5.

Interestingly, although the driving force of the grounded membrane configuration is nonlinear with displacement, it is linear with input voltage when the membrane is locked in its central resting position. This is due to the fact that the charges induced in the membrane's conductive coatings vary with the position of the membrane (which is also true for the single-ended configuration). Not surprisingly, it can be seen from Figs. 6 and 7 that the single-ended configuration yields a driving force that is neither linear with input voltage nor displacement.

XI. CONCLUSIONS

The force-versus-voltage and force-versus-displacement sensitivities of the electret transducer have been derived, assuming the charge to be evenly distributed throughout the membrane dielectric. Using the same derivation method, it can be shown that this is equivalent to having a concentrated charge layer in the middle. Thus, any symmetrical charge distribution can be represented by this model. Furthermore, this concentrated charge layer can be regarded as one plate of a capacitor, with the conductive coating forming the other. Hence, the external polarizing voltage for an equivalent nonelectret transducer is the same as that across this notional capacitor and is related to the charge by Eq. (39), which is essentially the total stored charge divided by the capacitance of the half membrane. The electromechanical force conversion factor has also been shown on an equivalent circuit, together with the interelectrode capacitance and negative capacitance, as a basis for simulation.

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